Bridge - MGF 3301 - Section 001

Homework 3

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be returned **by Wednesday February 5 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. [40 points total]

1.a) (10 points) Consider the following propositions:

$P =$"π is an irrational number."

$Q =$"$3 < 0$."

What is the truth value of $P \Rightarrow Q$? What about $Q \Rightarrow P$? What about $P \Leftrightarrow Q$? Justify your answers.

1.b) For each one of the conditional sentences here below, write its converse and its contrapositive:

(1.b1) (10 pts) "If it rains then I open the umbrella."

(1.b2) (10 pts) "If I am a farfalla then I am an insect."

(1.b3) (10 pts) "$x^2 + y^2 = 0 \Rightarrow x = 0$ and $y = 0$."

Ex 2. [30 points total] Consider the following open sentence:

$$P(x) = \text{“}0 < 3x + 1 \leq 10 \textit{ or } x \textit{ is a solution of } x^2 - 6x + 8 = 0.\text{”}$$

(a) (10 pts) What is the truth value of $P(4)$? What about $P\left(-\frac{1}{3}\right)$?

(b) (10 pts) If the Universe is $\mathbb{Z}$, what is the truth set of $P(x)$?

(c) (10 pts) If the Universe is $\mathbb{R}$, what is the truth set of $P(x)$?

Ex 3. [40 points total] Consider the following open sentence with Universe $\mathbb{Z}$:

$$P(n) = \text{“}n \textit{ is even and } n \textit{ is divisible by 6} \Rightarrow n \textit{ is divisible by 12.”}$$

Note that, for each n in $\mathbb{Z}$, $P(n)$ is a conditional sentence.

(a) (10 pts) What is the truth value of $P(3)$? What about $P(6)$? Justify your answers.

(b) (10 pts) What is the truth value of the statement "$\forall n$ in $\mathbb{Z}$, $P(n)$"? What about "$\exists\, n$ in $\mathbb{Z}$ such that $P(n)$"? Justify your answers.

(c) (10 pts) Write the contrapositive of $P(n)$ and the converse of $P(n)$.

(d) (10 pts) Consider the following definition:

> **Definition**
>
> An integer n is said to be **even** if and only if $\exists\, k$ in $\mathbb{Z}$ such that $n = 2k$.
>
> Given m and n in $\mathbb{Z}$, the integer n is said to be **divisible by** m if and only if $\exists\, k$ in $\mathbb{Z}$ such that $n = km$.

Use the above definition to prove that the **converse** of $P(n)$ is true for all integers n.