Bridge - MGF 3301 - Section 001

Homework 6

Instructions: Solve **Exercise 2 on this sheet** and all the other exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. This homework has to be returned **by Wednesday March 4 at 9:30 am**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Homework* component of the total grade (15%).

Ex 1. **[25 points total]** Describe the following sets with a *set-builder notation*, i.e. as truth set of an open sentence. In order to make this clear we provide the following example:

$$A = \{0, 3, 6, 9, 12, \ldots\} = \{n \in \mathbb{N} : \exists k \in \mathbb{Z} \text{ such that } n = 3k\}.$$

a) $A = \{0, 1, 4, 9, 16, 25, \ldots\}$

b) $B = \{-10, -5, 0, 5, 10, \ldots\}$

c) $C = \{\ldots, -8, -5, -2, 1, 4, 7, 10, \ldots\}$

d) $D = \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\right\}$

e) $E = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right\}$

Ex 2. **[20 points total]** Consider the following set:

$$X = \{\varnothing, 0, \{1, 2, 3\}, 3\}.$$

Determine the truth value of the following propositions:

$\varnothing \in X$	$\varnothing \subseteq X$	$\{\varnothing\} \subseteq X$	$0 \in X$	$\{0\} \subseteq X$
☐ TRUE	☐ TRUE	☐ TRUE	☐ TRUE	☐ TRUE
☐ FALSE	☐ FALSE	☐ FALSE	☐ FALSE	☐ FALSE

$\{0, 1\} \subseteq X$	$\{1, 2, 3\} \in X$	$\{1, 2, 3\} \subseteq X$	$\{\{1, 2, 3\}\} \subseteq X$	$X \subseteq X$
☐ TRUE	☐ TRUE	☐ TRUE	☐ TRUE	☐ TRUE
☐ FALSE	☐ FALSE	☐ FALSE	☐ FALSE	☐ FALSE

Ex 3. **[25 points total]** Let A, B, C and D be four nonempty sets such that:

- $C \subseteq A$;
- $\forall\, x \in C,\ x \in B$;
- $A \nsubseteq B$;
- $\exists\, y \in B$ such that $y \notin A$;
- $D \subseteq A$;
- $\forall\, z \in B,\ z \notin D$.

a) (12 points) Use Venn diagrams to display the above relationships among A, B, C and D.

b) (13 points) Build an example of four sets A, B, C, D that satisfy all the above conditions. (You may give this example by listing, for each set, all its elements.)

Ex 4. **[30 points total]** Consider the following sets:

$$A := \{x \in \mathbb{R} : x^3 - 9x^2 + 18x = 0\}$$
$$B := \{n \in 3\mathbb{Z} : n^2 - 14 \leq 5n\}$$
$$C := \{x \in \mathbb{R} : x^2 - 49 < 0\}$$

a) (10 points) Prove that $A \subseteq C$;

b) (10 points) Prove that $C \nsubseteq A$;

c) (10 points) Prove that $A = B$.

Ex 5. **[10 points total]** Prove (with a proof) or disprove (with a counterexample) the following statement (say clearly if you are proving or disproving it):

"If $A \subseteq B$ and $B \nsubseteq C$, then $A \nsubseteq C$."