Bridge - MGF 3301 - Section 001

Quiz 2 - Solution

01/22/2020

Instructions: The total number of points for this quiz is 10. You will get an extra point if you solve correctly the last exercise. Calculators are not allowed (and actually not needed).

EXERCISE 1
(10 points)

(a) [**1.5 point**] If P is true and $P \Rightarrow Q$ is true, what can we say about Q?

- ■ Q is true.
- ☐ Q is false.
- ☐ Q can be either true or false.
- ☐ This combination is not possible.

(b) [**1.5 point**] If P is true and $P \Rightarrow Q$ is false, what can we say about Q?

- ☐ Q is true.
- ■ Q is false.
- ☐ Q can be either true or false.
- ☐ This combination is not possible.

(c) [**1.5 point**] If P is false and $P \Rightarrow Q$ is false, what can we say about Q?

- ☐ Q is true.
- ☐ Q is false.
- ☐ Q can be either true or false.
- ■ This combination is not possible.

(d) [**1.5 point**] If Q is false and $P \Rightarrow Q$ is false, what can we say about P?

- ■ P is true.
- ☐ P is false.
- ☐ P can be either true or false.
- ☐ This combination is not possible.

(e) [**1.5 point**] If Q is true and $P \Rightarrow Q$ is true, what can we say about P?

- ☐ P is true.
- ☐ P is false.
- ■ P can be either true or false.
- ☐ This combination is not possible.

(f) **[1 points]** For one (and only one) among (a), (b), (c), (d) and (e), explain briefly your answer. (Please state which one among (a)-(e) you will discuss here below).

> **Solution**
>
> In order to answer correctly to any of the previous questions, it is enough to write down the truth table of the propositional form $P \Rightarrow Q$:
>
> | P | Q | $P \Rightarrow Q$ |
> |---|---|---|
> | T | T | T |
> | T | F | F |
> | F | T | T |
> | F | F | T |
>
> Indeed now we have simply to look at the row(s) that correspond to the combination of truth values given in each question, and return the truth value that the other component has on those rows.
>
> For instance in question (a), we know that P is true and $P \Rightarrow Q$ is true. There is only one row in the table that corresponds to this combination (the first one), and on that row the truth value of Q is true, which it the answer to (a).
>
> | P | Q | $P \Rightarrow Q$ |
> |---|---|---|
> | T | T | T |
> | T | F | F |
> | F | T | T |
> | F | F | T |

(g) **[1.5 points]** Anna says to Vanessa:

> *"If you come for dinner, I will cook either tagliatelle or tortellini."*

Knowing that Vanessa joined Anna for dinner and that Anna lied, which information do we have about the dishes proposed for dinner?

> **Solution**
>
> Let us set:
>
> $P :=$ "Vanessa joins Anna for dinner."
>
> $Q :=$ "Anna cooks either tagliatelle or tortellini." $=$ "Anna cooks tagliatelle or Anna cooks tortellini."

We know that P is true, while $P \Rightarrow Q$ is false. Then from the answer to the exercise 1.b we get that Q has to be false, or equivalently that $\sim Q$ has to be true. Using the Morgan's law, we have:

$\sim Q :=$ "Anna does not cook tagliatelle and Anna does not cook tortellini."

This means that we do not have neither tagliatelle nor tortellini among the dishes proposed for dinner (but who knows what she cooked, or if she did not cook at all!).

EXERCISE 2
(Bonus - 1 point)

Write a non-trivial denial (i.e. not of the form *It is not the case that...*) of the following sentence:

"*All students in* Bridge to Abstract Mathematics *are from the US.*"

Solution

A non-trivial denial of the above sentence is

"*There exists at least a student in* Bridge to Abstract Mathematics *that is not from the US.*"