TEST 3 - STUDY GUIDE

Bridge - MGF 3301 - Section 001

When? The third test will take place on **Wednesday April 22 at 9:30 am**.

Topics: **Sections 2.4, 3.1, 3.2, 3.3, 3.4, 4.1, 4.2, 4.3** of the textbook *A Transition to Advanced Mathematics*, by Smith, Eggen & St. Andre, 8th edition.

Office hours:

- Tuesday April 21: 5-7pm or by appointment

For the third test, you need to be able to:

- Prove by induction that a statement is true for all natural numbers (see Exercises 4 and 5 of The Last Homework).
- Prove/disprove that a relation is reflexive, symmetric or transitive (see Exercise 1 of Homework 9 and Exercise 2 of Homework 10).
- Prove that a relation R on a set A is an equivalence relation, describe the equivalence classes of specific elements and compute the set A modulo R (see Exercise 2 of Homework 10).
- Find the domain and range of a relation/function (see Exercise 3a of Homework 9).
- Prove that a family of sets is a partition of a given set (see Exercise 1 of Homework 10).
- Prove/disprove that a function is injective or surjective, find all the pre-images of a given element of the codomain of a function (see Exercises 1 and 2 of The Last Homework).

Moreover, you have to:

- Know all the definitions that appear in the next page and be able to apply them in the proof of other statements.

Review:

- Quiz 6 (and its solutions).
- Homework 8, 9, 10, 11 (and their solutions).
- Read again all the notes and/or Sections 2.4, 3.1, 3.2, 3.3, 3.4, 4.1, 4.2, 4.3 of the textbook.

Definitions

- The **domain** of a relation R from A to B is the set:
$$\mathrm{Dom}(R) = \{x \in A : \exists\, y \in B \text{ such that } (x, y) \in R\}.$$

- The **range** of a relation R from A to B is the set:
$$\mathrm{Rng}(R) = \{y \in B : \exists\, x \in A \text{ such that } (x, y) \in R\}.$$

- If R is a relation from A to B, then the **inverse** of R is the relation
$$R^{-1} = \{(y, x) : (x, y) \in R\}.$$

- Let R be a relation from A to B and S be a relation from B to C. The **composite** of R and S is the relation
$$S \circ R = \{(a, c) : \exists\, b \in B \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}.$$

- Let R be a relation on a set A.
  - R is **reflexive on A** if for all $x \in A$, $(x, x) \in R$.
  - R is **symmetric** if for all $x, y \in A$, $(x, y) \in R \Rightarrow (y, x) \in R$.
  - R is **transitive** if for all $x, y, z \in A$, $(x, y) \in R$ and $(y, z) \in R, \Rightarrow (x, z) \in R$.

- A relation R on a set A is an **equivalence relation on A** if R is reflexive on A, symmetric and transitive.

- Let R be an equivalence relation on a set A. For $x \in A$, the **equivalence class of x modulo R** is the set
$$\overline{x} = \{y \in A : (x, y) \in R\}.$$
The set
$$A/R = \{\overline{x} : x \in A\}$$
of all equivalence classes is called A **modulo** R.

- Let $m \in \mathbb{N}$. For $x, y \in \mathbb{Z}$ we say that x is **congruent to y modulo m** and write $x \equiv y \pmod{m}$ if $m \mid (x - y)$.

- We denote by $\mathbb{Z}_m$ the set of equivalence classes for the relation congruence modulo m. We have
$$\mathbb{Z}_m = \{\overline{0}, \overline{1}, \ldots, \overline{m-1}\},$$
where for $0 \leq r \leq m - 1$,
$$\overline{r} = \{mk + r : k \in \mathbb{Z}\}.$$

- Let A be a nonempty set. $\mathcal{P}$ is a **partition of** A if $\mathcal{P}$ is a set of subsets of A such that
  - $\forall\, X \in \mathcal{P}$, $X \neq \varnothing$.
  - $\forall\, X, Y \in \mathcal{P}$ such that $X \neq Y$, $X \cap Y = \varnothing$.
  - $\displaystyle\bigcup_{X \in \mathcal{P}} X = A$.

Definitions

- A **function from** A **to** B is a relation f from A to B such that
  - the domain of f is A and
  - if $(x, y) \in f$ and $(x, z) \in f$ then $y = z$.

  The set B is called the **codomain** of f. When $(x, y) \in f$, we write $f(x) = y$. In this casse we say that y is **the image of** f **at** x and that x is **a pre-image of** y.

- A function $f : A \to B$ is **onto B** (or is a **surjection**) if $\mathrm{Rng}(f) = B$, or equivalently if
$$\forall\, y \in B, \exists\, x \in A \text{ such that } f(x) = y.$$

- A function $f : A \to B$ is **one-to-one** (or is a **injection**) if
$$\forall x, y \in A,\ f(x) = f(y) \Rightarrow x = y,$$
  or equivalently if
$$\forall x, y \in A,\ x \neq y \Rightarrow f(x) \neq f(y).$$

- A function $f : A \to B$ is a **one-to-one correspondence** (or a **bijection**) if f is one-to-one and onto B.