Calculus I - MAC 2311 - Section 007

Homework - Review Test 1

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1) Compute the following limits (and show all your work):

a) $\lim_{x\to 0} \frac{x}{x^2+1}$

b) $\lim_{x\to -1} \frac{x+1}{x^2+3x+2}$

c) $\lim_{x\to 1} \frac{x^3-x^2+x-1}{x-1}$

d) $\lim_{x\to 4} \frac{-\sqrt{x}+2}{x-4}$

e) $\lim_{x\to 0} \frac{x}{\sqrt{2+x}-\sqrt{2-x}}$

f) $\lim_{x\to \infty} \frac{2x^5-x^3+3}{6x^5+1}$

g) $\lim_{x\to -\infty} \frac{x^3-x^2+x-1}{x-1}$

h) $\lim_{t\to \infty} \frac{t+1}{t^2+1}$

i) $\lim_{x\to -\infty} (x+\sqrt{3-x})$

j) $\lim_{x\to 2} \frac{x-3}{(x-2)^2}$

k) $\lim_{x\to 0} \frac{x^3-2}{x}$

l) $\lim_{\alpha\to 0} \frac{\sin(3\alpha)}{6\alpha}$

m) $\lim_{x\to \frac{\pi}{2}} \frac{\sin(x-\frac{\pi}{2})}{x-\frac{\pi}{2}}$

n) $\lim_{x\to 1^-} \frac{-|x-1|}{x-1}$

2) Sketch the graph of a function f which is defined for all real numbers and satisfies simultaneously the following:

a) $\lim_{x \to \infty} f(x) = 4$

b) The line $y = -1$ is a horizontal asymptote.

c) $f(0) = 1$.

d) The line $x = 2$ is a vertical asymptote.

e) $\lim_{x \to 2^+} f(x) = -\infty$

f) $x = 1$ is a solution for the equation $f(x) = 0$.

3) Let f be the function:

$$f(x) = \begin{cases} \frac{x}{x+1}, & x < -1; \\ x^2 + 2, & -1 \leq x \leq 2; \\ \cos(\pi x) + 5, & x > 2 \end{cases}$$

a) Compute $f(-1)$, $\lim_{x \to (-1)^-} f(x)$, $\lim_{x \to (-1)^+} f(x)$, $f(2)$, $\lim_{x \to 2^-} f(x)$, $\lim_{x \to 2^+} f(x)$.

b) Is the function f continuous at $x = -1$? And at $x = 2$?

4) State the Intermediate Value Theorem. Then, use it to prove that the equation:

$$x^2 + \sin\left(\frac{\pi}{2}x\right) + 2 = 3$$

has at least one solution in $[0, 1]$.

5) Write the equations of the vertical and horizontal asymptotes of the following function:

$$f(x) = \frac{3x^3 + 4x}{x^3 - 2x}.$$

6) Find the derivative (or the instantaneous rate of change) of the function $f(x) = \sqrt{x} + 1$ at the point $a = 4$. Then, write the equation of the tangent line to the curve $y = f(x)$ at the point $P(4, 3)$.