Calculus I - MAC 2311 - Section 007

Homework - Review Test 3

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Ex 1. (4+4+4+4+4 points) Compute the following limits. If you use l'Hospital's Rule state which type of indeterminate form you have.

a) $\lim\limits_{x\to\infty} \dfrac{\ln(1+x^2)}{x^2}$

b) $\lim\limits_{x\to 0} \dfrac{\sin(\pi e^x)}{x}$

c) $\lim\limits_{x\to\infty} \dfrac{e^{-x}+1}{x}$

d) $\lim\limits_{x\to 0^+} (e^x + x)^{\frac{1}{x}}$

e) $\lim\limits_{x\to\infty} x\left(\dfrac{\pi}{2} - \tan^{-1}(x)\right)$

Ex 2. (20 points) After their romantic dinner at the intersection of Bruce B. Downs and Fowler Avenue, the alligators from HW 2 decide to hold hands and take a walk along Fowler Avenue. Their position after t hours was

$$f(t) = \frac{\pi}{4} - \arctan((t-1)^2) \quad \text{miles.}$$

Which is the farthest point from the intersection reached by the alligators between 0 and 2 hours?

Ex 3. (2+3+2+4+4+5 points) Consider the function

$$f(x) = \frac{1}{x} + x + 1.$$

a) Find the domain of definition of f.

b) Find the horizontal and vertical asymptotes.

c) Find the critical numbers of f.

d) Find the intervals of increase/decrease of f and the local maxima/minima of f.

e) Find the intervals where f concaves upward/downward and the inflection points of f.

f) Sketch the graph of $y = f(x)$, by using the information you collected above.

Ex 4. (20 points) Among all boxes with a square base and volume 27 cm^3, what are the dimensions of the box which minimize the surface area?

Ex 5. (5+5+5+5 points) Which statements are True/False? Justify your answers.

a) We have $\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$ for all x in $[-1, 1]$.

b) If f is a function which is continuous on $[a, b]$, differentiable on (a, b) and such that $f(a) = f(b)$ then f has at least one critical point in (a, b).

c) There exists a function f such that $f(0) = 0$, $f(8) = 8$ and $f'(x) \geq 16$ for all x in $[0, 8]$.

d) If $f'(x) = g'(x)$ for all x in $\mathbb{R}$, then $f(x) = g(x)$.