Calculus I - MAC 2311 - Section 007

Quiz 1 - Solution

08/31/2017

1) Find the domain of the following function (justify each step):

$$f : \mathbb{R} \to \mathbb{R}$$

$$f(x) = \frac{x^3 + 2x}{x^2 - 3x + 2}$$

Solution:

Since $f(x)$ is a rational function, its domain consists of all the real numbers except those for which the denominator equals 0. Hence, the problem boils down to solve the quadratic equation:

$$x^2 - 3x + 2 = 0.$$

Its roots are $x_1 = \frac{3-\sqrt{1}}{2} = 1$ and $x_2 = \frac{3+\sqrt{1}}{2} = 2$. These are all the values for which the denominator is 0. Hence the domain is given by the set $D = \mathbb{R} \setminus \{1, 2\}$ - or we can write $D = (-\infty, 1) \cup (1, 2) \cup (2, \infty)$.

2) Find the limit (justify each step):

a) $\lim_{x \to -1} \frac{4x^2 + 1}{x^2}$

Solution:

Since $x = -1$ belongs to the domain of the function $f(x) = \frac{4x^2+1}{x^2}$ and a rational function is continuous at each point of its domain, then:

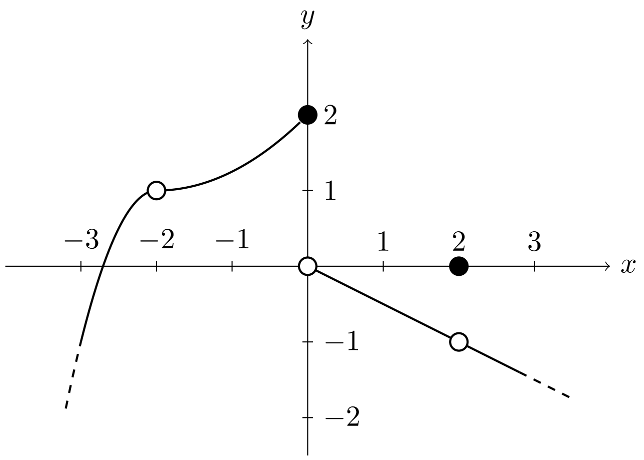
$$\lim_{x \to -1} f(x) = f(-1) = \frac{4 \cdot (-1)^2 + 1}{(-1)^2} = 5$$

b) $\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x}$

Solution:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} &= \lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \\
&= \lim_{x \to 0} \frac{\sqrt{x+1}^2 - 1^2}{x(\sqrt{x+1}+1)} = \\
&= \lim_{x \to 0} \frac{x}{x(\sqrt{x+1}+1)} = \\
&= \lim_{x \to 0} \frac{1}{(\sqrt{x+1}+1)} = \\
&= \frac{1}{\lim_{x \to 0}(\sqrt{x+1}+1)} = \frac{1}{2}
\end{aligned}$$

c) The graph of a function f is given.



Use it to evaluate the following, when it is possible:

a) $\lim_{x \to 0^-} f(x) = 2$

b) $\lim_{x \to 0^+} f(x) = 0$

c) $\lim_{x \to 0} f(x)$ *does not exist.*

Justify your answer: *Because* $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

d) $f(0) = 2$