Calculus I - MAC 2311 - Section 003

Review session Test 3

11/01/2018

Ex 1. Consider the function

$$f(x) = x \cdot e^x.$$

a) Find the critical numbers of f.

b) Find the intervals over which f is increasing/decreasing and the local maximum/minimum value of f.

c) Find the intervals where f is concave upward/downward and the inflection points of f.

Ex 2. Sketch the graph of **a** function f that satisfies **all of the given conditions**:

a) f is continuous on $(-\infty, \infty)$;

b) $\lim_{x\to-\infty} f(x) = 0$;

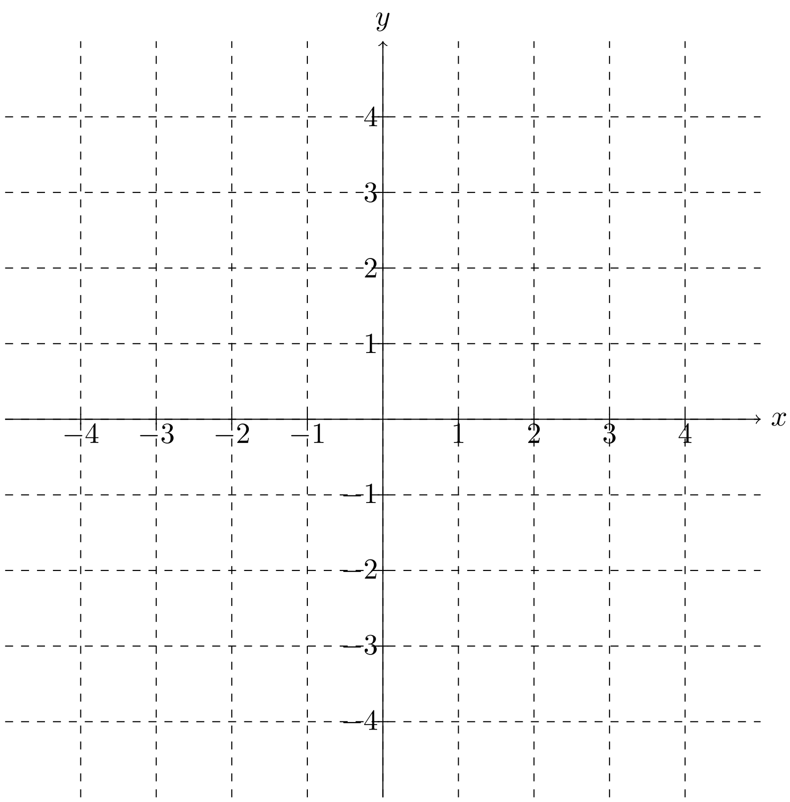
c) $f''(x) < 0$ on $(-\infty, -2)$.

d) $x = -2$ is the x-coordinate of an inflection point of f.

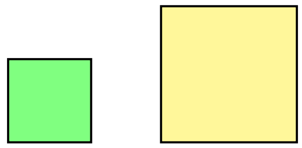
e) f has an absolute minimum at $x = -1$;

f) $f'(x) > 0$ on $(-1, \infty)$;

Make sure that your graph is the graph of a function, i.e. it passes the vertical line test.



Ex 3. A farmer has 400 feet of fencing and wants to fence two square fields, each one on all four sides (see the picture below). What are the length of the sides of the two square fields when they cover (together) the least area?



Ex 4. Find the absolute maximum and minimum values of the function

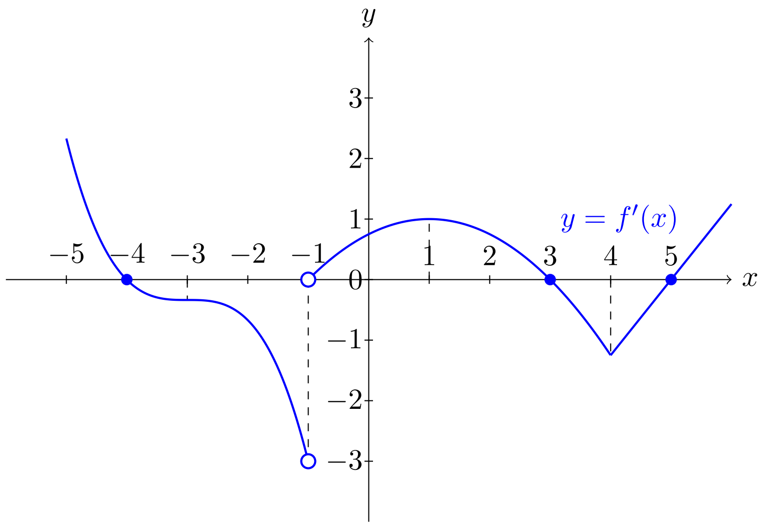
$$f(x) = 3x^4 - 4x^3 - 12x^2$$

on the closed interval $[-2, 1]$.

Ex 5. a) Let f be a differentiable function such that $f'(x) \geq -1$ for all x in $\mathbb{R}$. If $f(3) = -1$, what is the smallest value that f may attain at 5?

b) Prove that there does not exist a differentiable function such that $f(-3) = 0$, $f(1) = 2$ and $f'(x) \leq \frac{1}{3}$ for all $x \geq -5$.

Ex 6. The graph of the derivative f' of a **continuous** function f is shown below.



a) What are the critical numbers of f?

b) Over which intervals is the function f increasing/decreasing?

c) At what numbers does f have a local minimum/maximum value?

d) Over which intervals is f concave down/up?

e) What are the x-coordinates of the inflection points?

Ex 7. Let $f(x)$ be a differentiable function such that $f(x) > 0$ for all x. Let $H(x) = \ln(f(x))$. Prove that if x is a critical number for $H(x)$, then x is a critical number for $f(x)$.

Ex 8. Among all boxes with a square base and volume 27 cm^3, what are the dimensions of the box which minimize the surface area?

Ex 9. Which statements are True/False? Justify your answers.

a) The function $f(x) = \ln(x+1)$ has an absolute maximum and minimum value on $[-1, 1]$.

b) If f is a function such that $f''(x) > 0$ for all x, and $f'(1) = 1$, then the absolute maximum value of f on the interval $[1, 3]$ is $f(3)$.

c) The function
$$f(x) = \begin{cases} -x^2, & \text{when } x < 0 \\ x^2 + 1, & \text{when } x \geq 0 \end{cases}$$
has an inflection point at $(0, 1)$ since $f''(x) < 0$ on $(-\infty, 0)$ and $f''(x) > 0$ on $(0, \infty)$.

d) Let f be a function such that $f'(x) \neq 0$ for all x. Then the equation $f(x) = 0$ can have two different solutions x_1 and x_2.

e) If f is a function which is continuous on $[a, b]$, differentiable on (a, b) and such that $f(a) = f(b)$ then f has at least one critical point in (a, b).

f) There exists a function f such that $f(0) = 0$, $f(8) = 8$ and $f'(x) \geq 16$ for all x in $[0, 8]$.

g) If $f'(x) = g'(x)$ for all x in $\mathbb{R}$, then $f(x) = g(x)$.