Calculus I - MAC 2311 - Section 003

Review session Final Exam

11/29/2018

Ex 1. Compute the following (definite or indefinite) integrals:

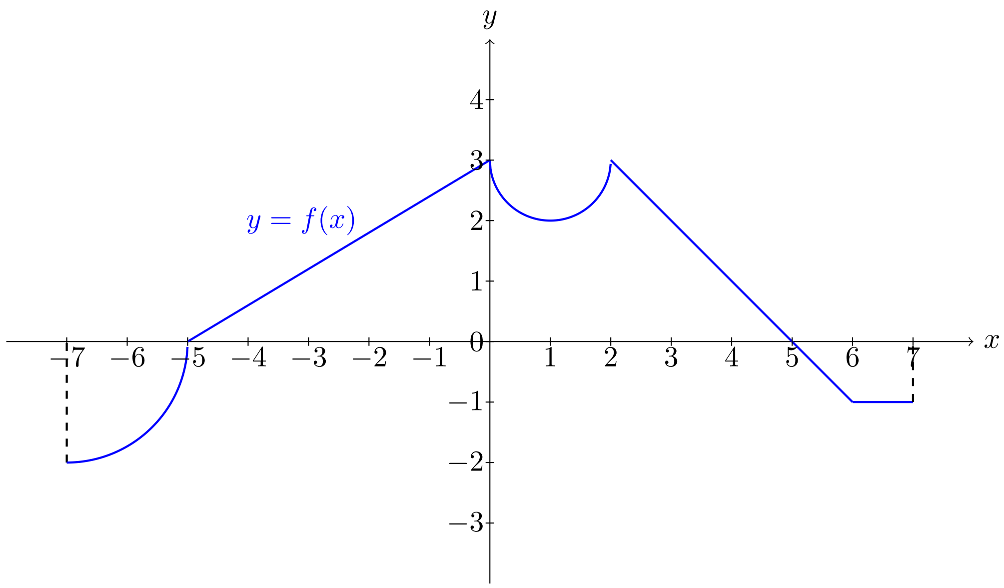
a) $\displaystyle\int 3\sin(x) + \frac{4}{1+x^2} + 2\,dx$

b) $\displaystyle\int (t+3)(2-t^2) + \frac{\sqrt{t}+t}{t^2}\,dt$

c) $\displaystyle\int_{-\pi}^{\frac{\pi}{2}} 3\sin(x) - 8\cos(x)\,dx$

d) $\displaystyle\int_{1}^{0} -2e^u + \frac{1}{1+u^2}\,du$

Ex 2. Let f be the function whose graph is the following:



a) Compute $\displaystyle\int_{-7}^{7} f(x)\,dx$.

b) Compute $\displaystyle\int_{-7}^{0} 3f(x)\,dx + \int_{0}^{5} f(x) + \sqrt{25-x^2}\,dx - \int_{7}^{5} 2f(x) + 2x\,dx$.

Ex 3. A ball is thrown upward at a speed of 48 feet per second from the edge of a cliff 288 feet above the ground.

a) Find its height above ground t seconds later.

b) When does it reach its maximum height?

c) When does it hit the ground?

Ex 4. Sketch the graph of **a** function f that satisfies **all of the given conditions**:

a) $\lim_{x\to-\infty} f(x) = 2$;

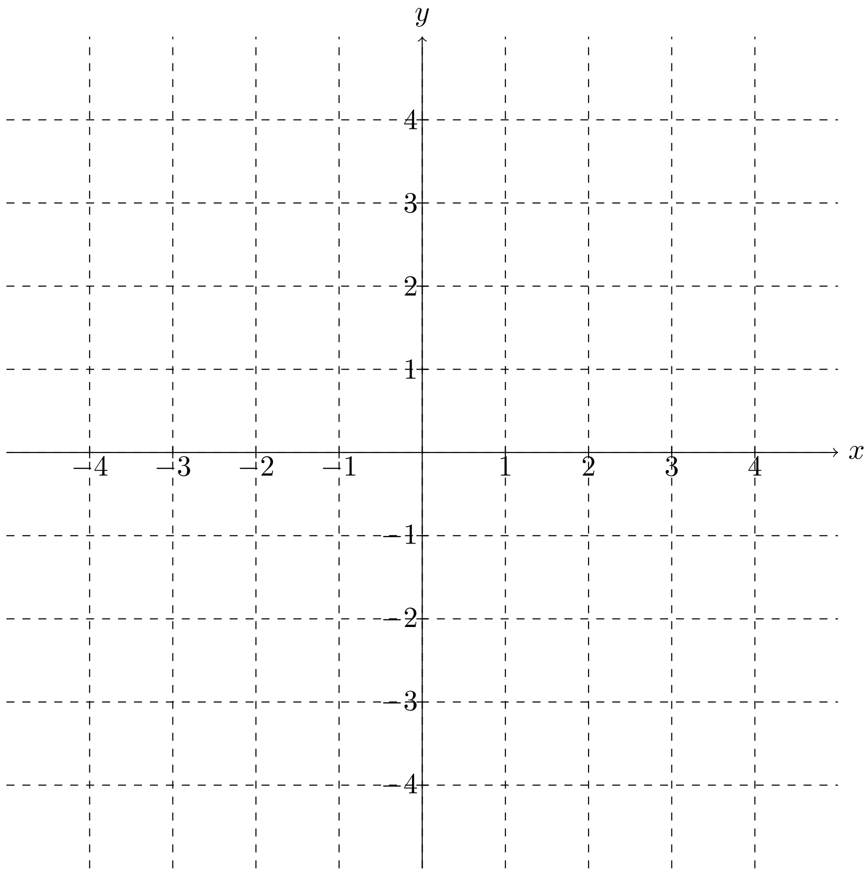
b) $f''(x) > 0$ for all $-3 < x < 1$;

c) $f'(-1) = 0$;

d) $f(2) = 1$;

e) $\int_2^x f(t)\,dt \geq 1$ for all $x > 3$.

Make sure that your graph is the graph of a function, i.e. it passes the vertical line test.



Ex 5. At noon ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 pm.

Ex 6. Compute the derivative of the following functions:

a) $f(t) = \sqrt{1 + t\arccos(t)}$

b) $f(x) = \dfrac{e^{\tan(x)} + 1}{\cos(x)}$

c) $f(s) = \arctan(\sqrt{s}) \cdot \ln(2s)$

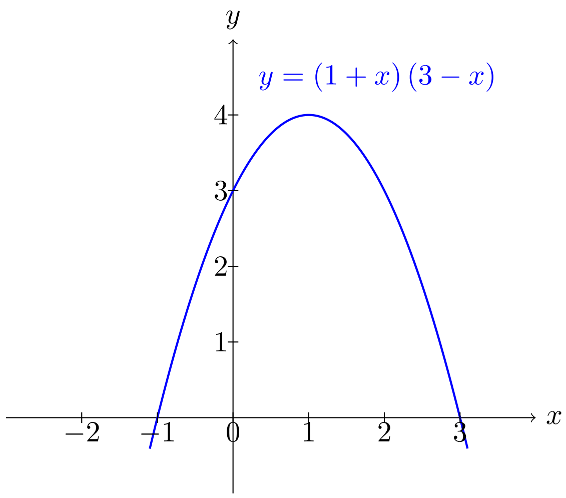
d) $g(x) = \int_{-1}^{x} e^t \cdot (t^2 - 3t + 2)dt$

What are the critical numbers of $g(x)$?

e) $g(t) = \int_{0}^{t^2} \frac{x-1}{x^2+1}dx$

What are the critical numbers of $g(t)$?

Ex 7. Consider the function $f(x) = (1+x)(3-x)$ whose graph on the interval $[-1, 3]$ is sketched below. Let S be the region between the curve $y = f(x)$, the x-axis and the lines $x = -1$ and $x = 3$.



a) Draw in the picture above the rectangles associate to the right Riemann sum with $n = 4$.

b) Approximate the area of S with the right Riemann sum with $n = 4$.

c) Express the area of S as a definite integral.

d) Compute the exact value of the area of S.

e) Was your approximation an underestimate or an overestimate?

Ex 8. Compute the following limits:

a) $\lim_{t\to 1} \frac{\ln(1+\ln(t))}{t^2-1}$

b) $\lim_{x\to 3} \frac{\sin(\frac{\pi}{2}x)}{\cos(\pi x)}$

c) $\lim_{x\to 0} \frac{e^{x^2}-1}{3x^2}$

d) $\lim_{x\to\infty} \int_{1}^{x} \frac{1}{1+t^2} + \frac{1}{t^2}dt$

Ex 9. Let $f(x) = \cos\left(\tan^{-1}\left(\frac{1}{e^x}\right)\right)$. Simplify the expression of f and compute $f(0)$.

Ex 10. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is as small as possible?

Ex.11 A particle is moving with acceleration given by the function $a(t) = 12t^2 + 2\sin(t)$ (measured in meters per second squared).

a) Find the position function of the particle if its initial velocity is 5 meters per second and the position at $t = \pi$ is π^4 meters.

b) Find the position function of the particle if its initial position is 2 meters and its position at $t = \frac{\pi}{2}$ is 0 meters.