# LOGIC IMPLICATION

# Recall

A function $f$ is *differentiable* at $a$ if $f'(a)$ exists, i. e. if

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists.

If $f$ is differentiable at $a$ then $f$ is continuous at $a$

$f$ is differentiable at $a$

$$\Downarrow$$

$f$ is continuous at $a$

$$P \Rightarrow Q$$

P = « $f$ is differentiable at $a$ »

Q = « $f$ is continuous at $a$ »

$$P \Rightarrow Q$$

$P$ = Student X is in CHE 217 on MW at 12:30pm

$Q$ = Student X is a calculus student

Is this implication true?

**YES!**

# Question:

If P $\Rightarrow$ Q is true, then what can we say about:

not Q $\Rightarrow$ not P (contrapositive)

Q $\Rightarrow$ P (converse)

P = Student X is in CHE 217 on MW at 12:30

Q = Student X is a calculus student

not P = Student X is **not** in CHE 217 on MW at 12:30

not Q = Student X is **not** a calculus student

Is it true that: not Q $\Longrightarrow$ not P ?

**Yes!**

P = Student X is in CHE 217 on MW at 12:30
Q = Student X is a calculus student

Is it true that: $Q \Rightarrow P$ ?

**NO!**

**Counterexample:** each student in sections 2,4,5,6,7,etc. of calculus is a calculus student who is not in CHE 217 on MW at 12:30pm.

# Another example

$\Longrightarrow$ insect

**TRUE**

# Another example

not insect $\Rightarrow$ not 

**TRUE**

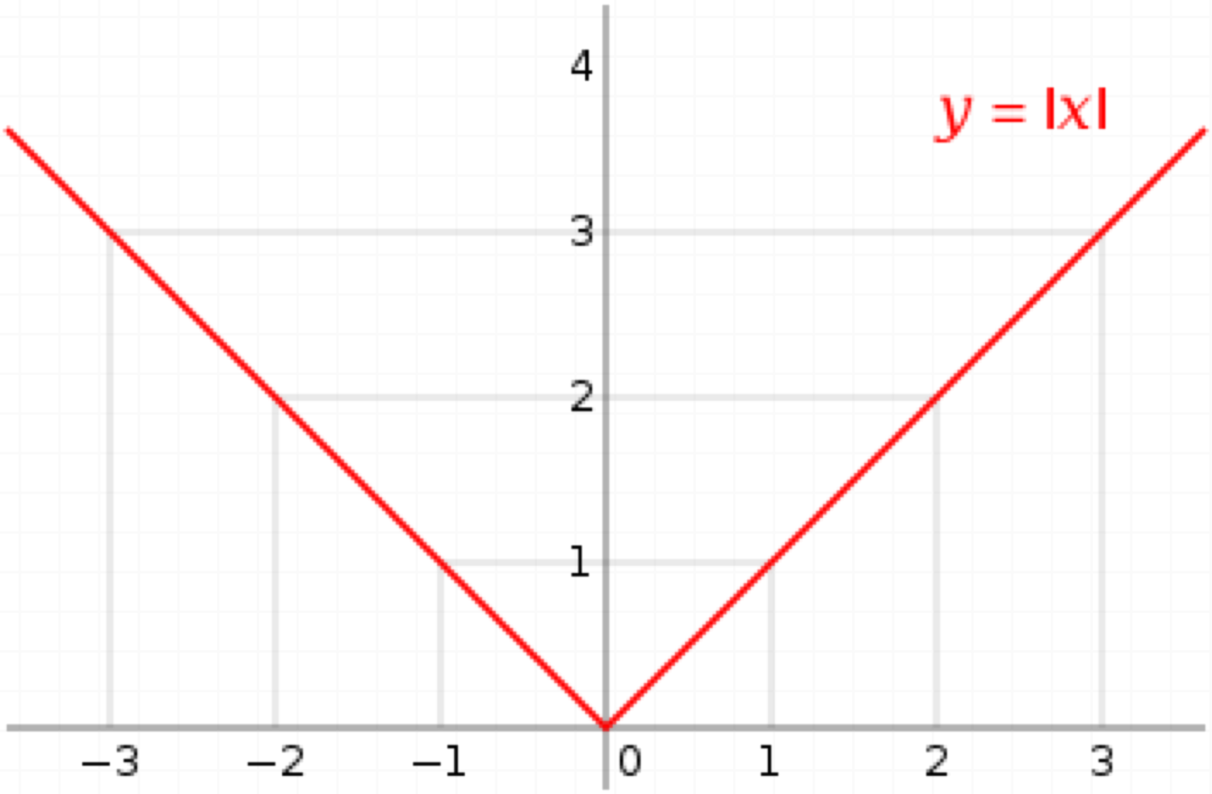
# Another example

insect $\Rightarrow$ 

**FALSE!**

counterexample

# Counterexample




$f(x) = |x|$ is continuous at 0, but not differentiable at 0.

# Recap!

The implication **P** $\Rightarrow$ **Q** is true when every time the statement P is true, then also the statement Q is true. Hence:

- If you want to show that the implication P $\Rightarrow$ Q is **true**, you need a **proof**;
- If you want to show that the implication P $\Rightarrow$ Q is **false** you need a **counterexample**: this means that you need an example of something that verifies P but does not verify Q (indeed in this case P will be true, while Q will be false).

# Now it's your turn!

Let $n$ be an integer.

Consider the following implication:

If $n$ is even then $n^2$ is even.

Is it true? **Yes!**

# Now it's your turn!

Let $n$ be an integer.

Consider now the **converse** of the previous implication:

If $n^2$ is even then $n$ is even.

Is it true? **Yes**

# An example of double implication

We have proven that:

For all integers $n$,

$n^2$ is even **if and only if** $n$ is even.

$$P \Leftrightarrow Q$$

P = The final grade of student X is A

Q = The final grade of student X is more than 90%

# All the definitions are « *if and only if* »

Ex: A function $f$ is *continuous* at $a$ if (and only if)

$$\lim_{x \to a} f(x) = f(a)$$