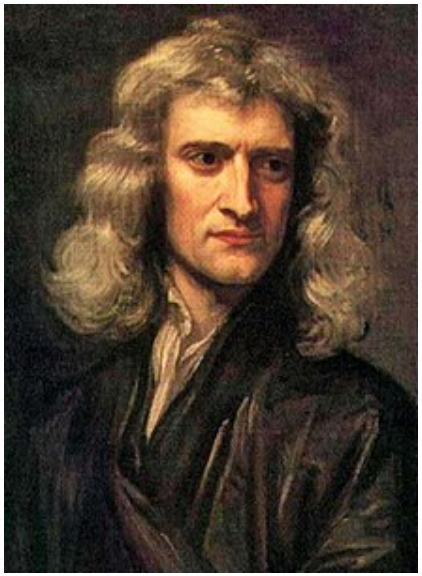
# INTRODUCTION TO CALCULUS

# ETYMOLOGY

The word **calculus** comes from Latin and means *« a small pebble or stone used for counting »*



# BRIEF HISTORY

Modern calculus was developed in the 17th century by Newton and Leibniz indipendently of each other.

**Newton**: first to apply calculus to general physics.

**Leibniz**: developed much of the notation used in calculus today

# What does calculus study?

Calculus is the study of the **change**, and it studies change by studying *istantaneous* change (over a tiny interval of time).

**Example:** Motion of an object along a fixed path

# Motion of an object along a fixed path

- Let us fix a point on the path. At any time we can describe the position as the distance (with positive or negative sign) of the object from the fixed point: position is a *function* of time.

- What does it change in this example?
  The position varies with time.

- And how does the position change with time?
  This depends on the *velocity* of the object.

# Average velocity…


Sam and Alex are traveling in the car … but the speedometer is broken.

Alex: "Hey Sam! How fast are we going now?"

Sam: "Wait a minute …"

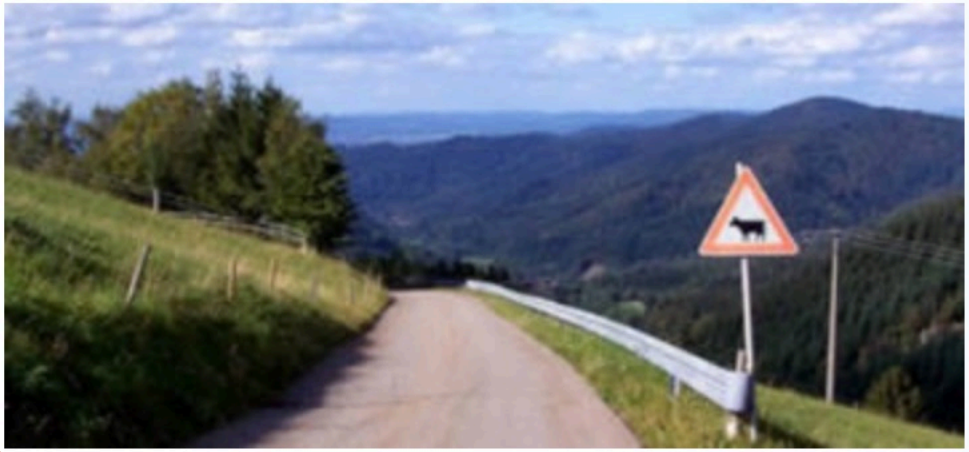
"Well in the last minute we went 1,2 km, so we are going:"

1,2 km per minute x 60 minutes in an hour = **72 km/h**

Alex: "No, Sam! Not our **average** for the last minute, or even the last second, I want to know our speed RIGHT NOW."

# … vs instantaneous velocity

Sam: "OK, let us measure it up here … at this road sign… NOW!"



Here we need limits!!

"OK, we were AT the sign for **zero seconds**, and the distance was … **zero meters**!"

The speed is 0m / 0s = 0/0 = **I Don't Know**!

"I can't calculate it Sam! I need to know **some** distance over **some** time, and you are saying the time should be zero? Can't be done."

# Two problems

1) Find the instantaneous velocity given a position (we will see that more in general this corresponds to compute the *derivative* of a function)

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**DIFFERENTIAL CALCULUS**

2) Find the position function by knowing the istantaneous velocity at all time (or, more in general, find the function by knowing its derivative).

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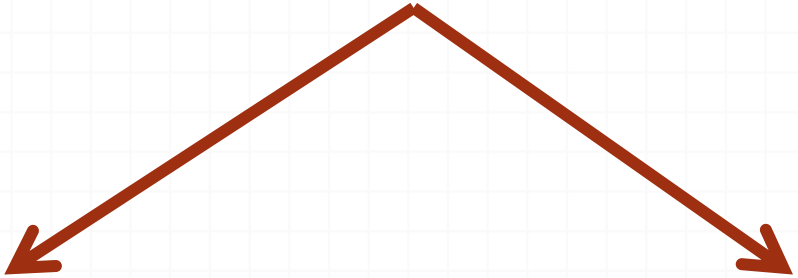
**INTEGRAL CALCULUS**

# CALCULUS



## DIFFERENTIAL CALCULUS

## INTEGRAL CALCULUS

## FUNDAMENTAL THEOREM OF CALCULUS