Calculus I - MAC 2311 - Section 001

Homework 1

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. The homework has to be returned **by Wednesday January 31, 11 am**. The total number for this homework is 104 (there are 4 extra points). The grade you will receive for this homework will count as a part of *Quizzes and handwritten homework* component of the total grade (15%).

Ex 1. (24 points) Compute the following limits and show all your work:

a) $\lim_{x\to-\sqrt{2}} \dfrac{x^2}{x+1}$

b) $\lim_{t\to-1} \dfrac{t^2-1}{t^2+7t+6}$

c) $\lim_{x\to1} \dfrac{-\sqrt{x}+1}{2x-2}$

d) $\lim_{x\to\infty} \dfrac{2017x^{2017}+2017}{2018x^{2018}+2018}$

e) $\lim_{x\to-\infty} \dfrac{-3x^3+8x-1}{2x^3-x^2+4}$

f) $\lim_{u\to-\infty} \dfrac{u^2+u+1}{-u+1}$

g) $\lim_{\alpha\to0} \dfrac{\sin(8\alpha)}{2\alpha}$

h) $\lim_{\theta\to\frac{\pi}{2}^-} \dfrac{\sin x}{\cos x}$

i) $\lim_{x\to0} \dfrac{x-1}{x}$

j) $\lim_{x\to\infty} \dfrac{1}{x+\sqrt{3+x}}$

k) $\lim_{x\to1} f(x)$, where $f(x) = \begin{cases} x^3-5x+7, & \text{when } x \le 1 \\ \sqrt{x+3}+1 & \text{when } x > 1 \end{cases}$

l) $\lim_{\alpha\to\frac{\pi}{2}} \dfrac{\sqrt{1-\cos(\alpha)}-\sqrt{1+\cos(\alpha)}}{\cos(\alpha)}$

Ex 2. (20 points) Sketch the graph of a function f which satisfies simultaneously the following conditions:

a) $\lim_{x\to\infty} f(x) = -2$,

b) The line $y = 3$ is a horizontal asymptote,

c) $f(3) = -3$,

d) The line $x = -1$ is a vertical asymptote,

e) $\lim_{x\to -1^+} f(x) = \infty$,

f) $\lim_{x\to -1^-} f(x) = 1$,

g) $x = -1$ is a solution for the equation $f(x) = 1$,

h) f has a removable discontinuity at $x = -3$.

Ex 3. (20 points) Let a and b be two constants (= two real numbers) and f be the function:

$$f(x) = \begin{cases} x^2 - 3x + a, & \text{when } x < -1 \\ 2\cos(\pi x), & \text{when } -1 \le x \le 2 \\ \frac{-2x+2b^2}{x}, & \text{when } x > 2. \end{cases}$$

a) Compute $f(-1)$, $\lim_{x\to(-1)^-} f(x)$, $\lim_{x\to(-1)^+} f(x)$, $f(2)$, $\lim_{x\to 2^-} f(x)$, $\lim_{x\to 2^+} f(x)$.

b) Find the values of a and b that make f continuous everywhere.

Ex 4. (20 points)

a) It is the Sunday before the test. A calculus student, following the suggestion of his instructor, decides to go hiking on the highest mountain in Florida in order to understand the Intermediate Value Theorem in a more concrete situation.
Let $h(t)$ be the function that at each time t (in hours) represents the height of the student above sea level (in feet). If

$$h(t) = -t^2 + 5t + 1,$$

prove that there is a time between 0 and 3 hours at which the student is 6 feet above sea level.

b) Compute the instantaneous rate of change of $h(t)$ at $t = 1$, that is $h'(1)$, by using the definition of derivative.

Ex 5. (20 points) Which statements are True/False? Justify your answers.

a) A function can have at most 2 horizontal asymptotes.

b) If $f(x) = \frac{P(x)}{Q(x)}$ is a rational function and a is a number such that $Q(a) = 0$ then $x = a$ is a vertical asymptote for f.

c) If $s(t)$ is a position function and $s(3) = 0$, then the velocity at $t = 3$ is zero.

d) If $-|x-1| \le f(x) \le |x-1|$ near 1, then $\lim_{x\to 1} f(x) = 0$.