Calculus I - MAC 2311 - Section 001

Quiz 4 - Solutions

02/14/2018

1) [10 points] For each of the following functions compute its derivative:

a) $f(x) = x^7 - 3x^2 - \dfrac{2}{x} + \sqrt[6]{x^5}$

Solution:

$$\begin{aligned} f'(x) &= \left(x^7 - 3x^2 - \frac{2}{x} + \sqrt[6]{x^5}\right)' = \\ &= (x^7)' - (3x^2)' - \left(2x^{-1}\right)' + \left(x^{\frac{5}{6}}\right)' = \\ &= (x^7)' - 3(x^2)' - 2\left(x^{-1}\right)' + \left(x^{\frac{5}{6}}\right)' = \\ &= 7x^6 - 3 \cdot 2x - 2 \cdot (-1) \cdot x^{-2} + \frac{5}{6} \cdot x^{\frac{5}{6}-1} = \\ &= 7x^6 - 6x + \frac{2}{x^2} + \frac{5}{6\sqrt[6]{x}}. \end{aligned}$$

b) $f(x) = x^3 \tan(x)$

Solution:

$$\begin{aligned} f'(x) &= \left(x^3 \tan(x)\right)' = \\ &= (x^3)' \cdot \tan(x) + x^3 \cdot (\tan(x))' = \\ &= 3x^2 \tan(x) + x^3 \sec^2(x). \end{aligned}$$

c) $f(x) = \cos\left(x^2 + 4\sin(x)\right)$

Solution:

$$\begin{aligned} f'(x) &= \left(\cos\left(x^2 + 4\sin(x)\right)\right)' = \\ &= -\sin\left(x^2 + 4\sin(x)\right) \cdot \left(x^2 + 4\sin(x)\right)' = \\ &= -\sin\left(x^2 + 4\sin(x)\right) \cdot \left(2x + 4\cos(x)\right). \end{aligned}$$

d) $f(x) = \dfrac{3 + \sin(2x)}{x^2 + 4}$

Solution:

$$\begin{aligned} f'(x) &= \left(\frac{3 + \sin(2x)}{x^2 + 4}\right)' = \\ &= \frac{(3 + \sin(2x))'(x^2 + 4) - (3 + \sin(2x))(x^2 + 4)'}{(x^2 + 4)^2} = \\ &= \frac{2\cos(2x)(x^2 + 4) - (3 + \sin(2x)) \cdot 2x}{(x^2 + 4)^2}. \end{aligned}$$

e) $f(x) = \sqrt{\cos\left(\frac{1}{x}\right)}$

Solution:

$$\begin{aligned} f'(x) &= \left[\sqrt{\cos\left(\frac{1}{x}\right)}\right]' = \\ &= \left[\left(\cos\left(\frac{1}{x}\right)\right)^{\frac{1}{2}}\right]' = \\ &= \frac{1}{2}\left(\cos\left(\frac{1}{x}\right)\right)^{-\frac{1}{2}} \cdot \left[\cos\left(\frac{1}{x}\right)\right]' = \\ &= \frac{1}{2}\left(\cos\left(\frac{1}{x}\right)\right)^{-\frac{1}{2}} \cdot \sin\left(\frac{1}{x}\right) \cdot \left[\frac{1}{x}\right]' = \\ &= \frac{1}{2}\left(\cos\left(\frac{1}{x}\right)\right)^{-\frac{1}{2}} \cdot \sin\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right). \end{aligned}$$

2) [2 points] State the Intermediate Value Theorem.

Theorem *(Intermediate Value Theorem). Let f be a continuous function on a closed interval $[a, b]$, with $f(a) \neq f(b)$. Then for every number N between $f(a)$ and $f(b)$ there exists a number c in (a, b) such that $f(c) = N$.*

3) [Bonus] Use the definition of $\cot(x)$ and the appropriate rule to show that the derivative of $\cot(x)$ is $-\csc^2(x)$ $\left(\text{or equivalently } -\frac{1}{\sin^2(x)}\right)$.

Solution:

Recall that $\cot(x) = \frac{\cos(x)}{\sin x}$. Then:

$$\begin{aligned} (\cot(x))' &= \left(\frac{\cos(x)}{\sin(x)}\right)' = \\ &= \frac{(\cos(x))'\sin(x) - \cos(x)(\sin(x))'}{\sin^2(x)} = \\ &= \frac{-\sin(x)\cdot\sin(x) - \cos(x)\cdot\cos(x)}{\sin^2(x)} = \\ &= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \\ &= \frac{-(\sin^2(x) + \cos^2(x))}{\sin^2(x)} = \\ &\overset{\sin^2(x)+\cos^2(x)=1}{=} -\frac{1}{\sin^2(x)} = -\csc^2(x). \end{aligned}$$