Name and surname:

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Calculus I - MAC 2311 - Section 001

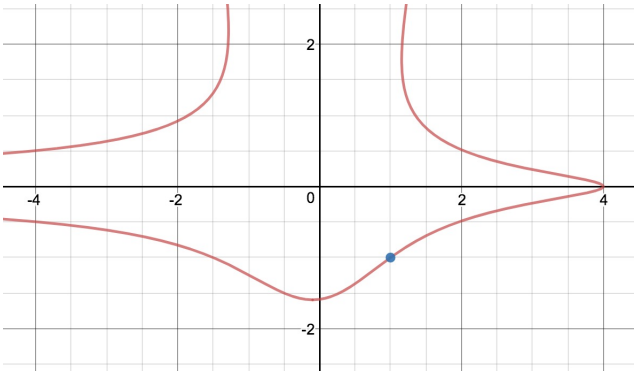
Quiz 5

02/21/2018

Instructions: The total number of points of this quiz is 10. You will get an extra point if you solve correctly the last exercise.

1) [5 points] Consider the curve $\mathcal{C}$ given by the equation

$$x - y^3 = 4 - 2x^2y^2.$$



a) Use implicit differentiation to find y' $\left(\text{ i.e. } \dfrac{dy}{dx}\right)$.

b) Find an equation of the tangent line to the above curve at the point $(1, -1)$.

2) [5 points] In thermodynamics, **Boyle's law** states that for a fixed amount of an ideal gas kept at a fixed temperature, pressure P and volume V are inversely proportional, i.e.

$$PV = k,$$

where k is a constant. Assume that the quantities P and V depend both on time.

a) Differentiate both sides of Boyle's law to find an equation relating $\frac{dP}{dt}$ and $\frac{dV}{dt}$.

b) A sample of gas is trapped in a cylinder by a piston which is slowly compressed. Suppose that at a certain instant the gas occupies a volume of 60 L (liters) and has a pressure of 50 kPa (kilopascal) and the volume of the gas decreases at a rate of 10 L/min. Assuming the temperature is constant, how quickly is the pressure increasing at this instant?

3) [Bonus] Compute the following derivative:

$$\frac{d}{du}\left[\tan(k^3 u)\right],$$

where k is a constant.