Calculus I - MAC 2311 - Section 001
Review session - Test 1
02/01/2018

Ex 1. Sketch the graph of a function f which satisfies **all** the following conditions:

    a) $\lim\limits_{x \to -\infty} f(x) = 2$,

    b) $f(-2) = 3$,

    c) $\lim\limits_{x \to 1^-} f(x) = -\infty$,

    d) $f(1) = 0$

    e) $\lim\limits_{x \to 1^+} f(x) = 0$,

    f) $\lim\limits_{x \to \infty} f(x) = -1$,

Ex 2. An alligator moves according to the position function $s(t) = t^2 - 4t - 1$, where position is measured in meters and time in seconds.

    a) Prove that between 0 and 5 seconds there is a time t_0 at which $s(t_0) = 0$.
    b) Find the instantaneous velocity $v(t)$ at each time t, by using the definition of derivative. (Recall that $v(t) = s'(t)$).
    c) What is the velocity of the alligator at $t = 5$ seconds?
    d) At what time is the velocity of the alligator zero?

Ex 3. Let f be the piecewise function defined as:

$$f(x) = \begin{cases} x^3 - 2cx - 2, & \text{when } x < -1; \\ c^2 \cdot \cos(-\pi x), & \text{when } x \geq -1. \end{cases}$$

Find the value(s) of c that make the function continuous everywhere.

Ex 4. Consider the rational function:

$$f(x) = \frac{-2x^2 + 2x + 12}{x^2 + 3x + 2}.$$

    a) Find the domain of $f(x)$.
    b) Compute $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$.
    c) Write the equation of the horizontal asymptote(s) of $f(x)$.
    d) Write the equation of the vertical asymptote(s) of $f(x)$.

Ex 5. Find the derivative of the function $f(x) = \sqrt{x} + x$. Then, write the equation of the tangent line to the curve $y = f(x)$ at the point $P(4, 6)$.

Ex 6. Compute the following limits:

a) $\lim\limits_{x\to 0} \dfrac{x}{x^2+1}$

b) $\lim\limits_{x\to -7} \dfrac{x+7}{x^2+6x-7}$

c) $\lim\limits_{t\to 3} \dfrac{\sqrt{3t}-3}{t^2-3t}$

d) $\lim\limits_{x\to 0} \dfrac{x}{\sqrt{2+x}-\sqrt{2-x}}$

e) $\lim\limits_{\theta\to 0} \dfrac{4\sin(17\theta)}{3\theta}$

f) $\lim\limits_{x\to\infty} \dfrac{2x^5-x^3+3}{6x^5+1}$

g) $\lim\limits_{x\to -\infty} \dfrac{x^3-x^2+x-1}{1-x}$

h) $\lim\limits_{t\to\infty} \dfrac{t+1}{t^2+1}$

i) $\lim\limits_{x\to 2} \dfrac{x-3}{(x-2)^2}$

j) $\lim\limits_{x\to 0^+} \dfrac{\sin(x+\frac{\pi}{2})+1}{x}$

k) $\lim\limits_{x\to 1^-} \dfrac{-|x-1|}{x-1}$

l) $\lim\limits_{\theta\to 0} \dfrac{\sin(2017\theta)}{\sin(2018\theta)}$

Ex 7.

Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.

(a)

(b)

(c)

(d)

I

II

III

IV