Calculus I - MAC 2311 - Section 001

Review session Test 3

4/06/2018

Ex 1. Giovanni has a budget of \$6000 for building a swimming pool with a square floor. On the floor he wants to use tiles that cost \$20 per square feet and on the sides he wants to use tiles that cost \$10 per square feet. What is the depth of the swimming pool with the largest volume he can construct with his budget?

Ex 2. Consider the function

$$f(x) = x\ln(x^2).$$

a) Find the domain of definition of f.

b) List the x- and y- intercept(s).

c) Find the horizontal and vertical asymptotes.

d) Find the critical numbers of f.

e) Find the intervals over which f is increasing/decreasing and the local maximum/minimum value of f.

f) Find the intervals where f is concave upward/downward and the inflection points of f.

g) Sketch the graph of $y = f(x)$, by using the information you collected above.

Ex 3. Find the absolute maximum and minimum values of the function

$$f(x) = 3x^4 - 4x^3 - 12x^2$$

on the closed interval $[-2, 1]$.

Ex 4. a) Simplify the expression $\cos\left(\sin^{-1}(x+1)\right)$.

b) Simplify the expression $\tan\left(\arccos(e^x)\right)$.

c) Compute the derivative of $\arcsin(e^x + x)$.

d) Compute the derivative of $\arccos(2x) \cdot \arctan\left(\frac{1}{x}\right)$.

Ex 5. a) Let f be a differentiable function such that $f'(x) \geq -1$ for all x in $\mathbb{R}$. If $f(3) = -1$, what is the smallest value that f may attain at 5?

b) Prove that there does not exist a differentiable function such that $f(-3) = 0$, $f(1) = 2$ and $f'(x) \leq \frac{1}{3}$ for all $x \geq -5$.

Ex 6. Compute the following limits. If you use l'Hospital's Rule state which type of indeterminate form you have.

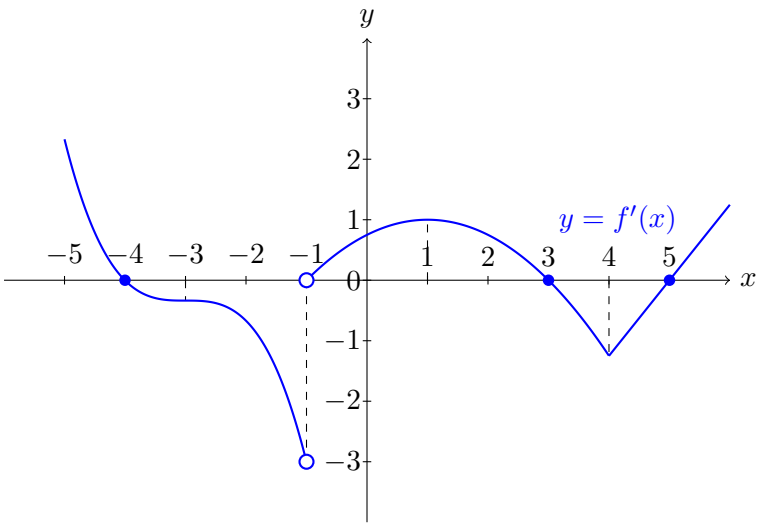
a) $\lim_{x\to\infty} \dfrac{e^{x+1}}{2x^2}$

b) $\lim_{x\to 0^+} \dfrac{\sin(x) - \ln(\cos(x))}{x^2}$

c) $\lim_{x\to 3} \dfrac{x^2 - 2x - 3}{x^2 - 3}$

d) $\lim_{x\to 0^+} \left(x^2\right)^{3x}$

Ex 7. The graph of the derivative f' of a continuous function f is shown below.



a) What are the critical numbers of f?

b) Over which intervals is the function f increasing/decreasing?

c) At what numbers does f have a local minimum/maximum value?

d) Over which intervals is f concave down/up?

e) What are the x-coordinates of the inflection points?