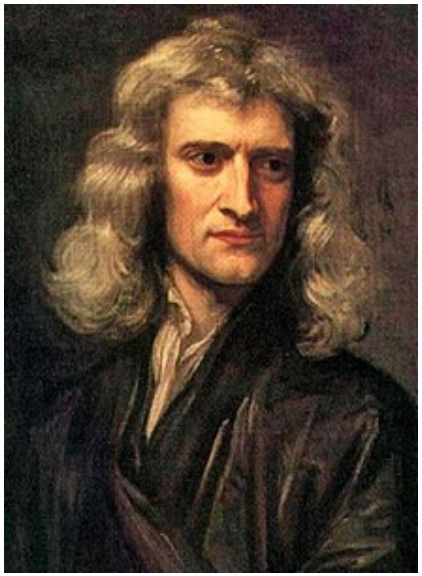
INTRODUCTION TO CALCULUS

ETIMOLOGY

The word **calculus** comes from Latin and means *« a small pebble or stone used for counting »*



BRIEF HISTORY

Modern calculus was developed in 17th-century by **Newton** and **Leibniz** indipendently of each other (even if there was a great controversy.

Newton: first to apply calculus to general physics.

Leibniz: developed much of the notation used in calculus today

What does calculus study?

Calculus is the study of the **change** and it studies change by studying *istantaneous* change (over a tiny interval of time).

Example: Motion of an object along a fixed path

Motion of an object along a fixed path

- Let us fix a point on the path. At any time we can describe the position (= distance frome the fixed point) of the object: position is a *function* of the time.

- What does it change in this example?
  The position varies with time.

- And how does the position change with time?
  This depends on the *velocity* of the object.

Average velocity…

Sam and Alex are traveling in the car … but the speedometer is broken.

Alex: "Hey Sam! How fast are we going now?"

Sam: "Wait a minute …"

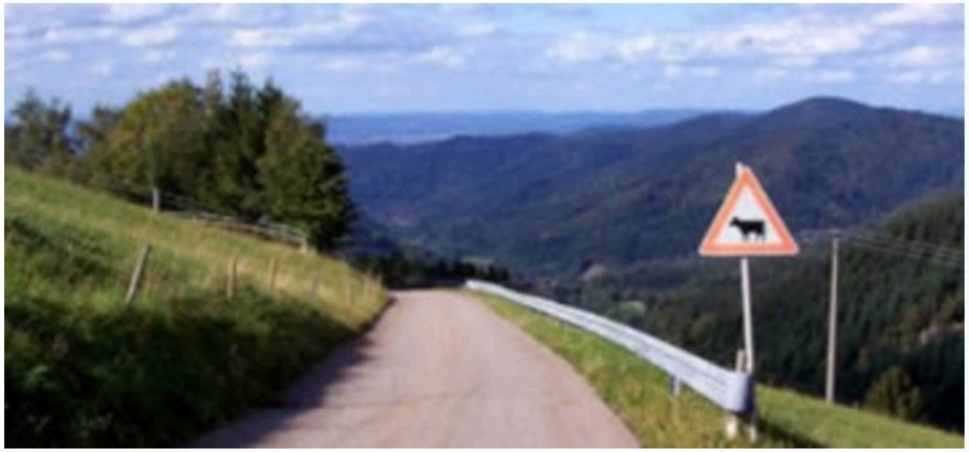
"Well in the last minute we went 1,2 km, so we are going:"

1,2 km per minute x 60 minutes in an hour = **72 km/h**

Alex: "No, Sam! Not our **average** for the last minute, or even the last second, I want to know our speed RIGHT NOW."

... vs instantaneous velocity

Sam: "OK, let us measure it up here ... at this road sign... NOW!"



"OK, we were AT the sign for **zero seconds**, and the distance was ... **zero meters**!"

The speed is 0m / 0s = 0/0 = **I Don't Know**!

"I can't calculate it Sam! I need to know **some** distance over **some** time, and you are saying the time should be zero? Can't be done."

Two problems

1) Find the instantaneous velocity by knowing the function position (called more in general the *derivative* of the function)

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DIFFERENTIAL CALCULUS

2) Find the position by knowing the istantaneous velocity at all time (or more in general, find the function by knowing its derivative).

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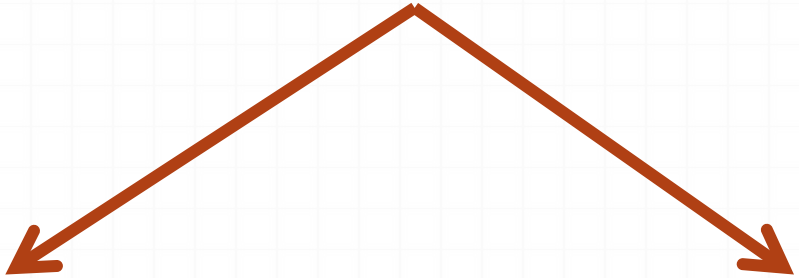
INTEGRAL CALCULUS

CALCULUS



DIFFERENTIAL CALCULUS

INTEGRAL CALCULUS

FUNDAMENTAL THEOREM OF CALCULUS